3 Network Stability

Stability resembles intelligence: easy to imagine and hard to define. Here I will assess network stability at three different levels. First I will describe network perturbations and the concept of local dissipation and global connections. Later a number of different scenarios will be shown, where the perturbation is big or persistent enough to change the stability of the underlying bottom networks, and therefore, the structure of the current network is not preserved. Then stability relationships between the bottom and top networks will be discussed using the example of synchrony. Finally, I will describe the two basic design alternatives: evolution and engineering. Both of them respond to the same question, as does the whole chapter: How can we make a network stable?[1]

If a network has violently changing properties, it is most probably not very stable. How can we measure stability, if a network remains unchanged? The assessment of stability often requires a test, and this test comes in the form of a perturbation to the network. A stable network should try to restore its original status after a perturbation. However, this is not easy. Most networks are open systems and therefore undergo a continuous series of perturbations. In the next section, I will give a survey of such perturbations.

3.1 Perturbations. Good and Bad Noise

Perturbations are often regarded as noise. What is the difference? Noise is usually understood from the point of the experimenter. If we measure it from the outside, noise is the fluctuation of the value we measure. However, from the point of view of the network, noise is a series of perturbations changing its original status. Network perturbations can be called either signals or noise. This dissection is rather artificial and

[1] If you need a definition of stability, let me encourage you to jump to Sect. 4.3. But please come back again!

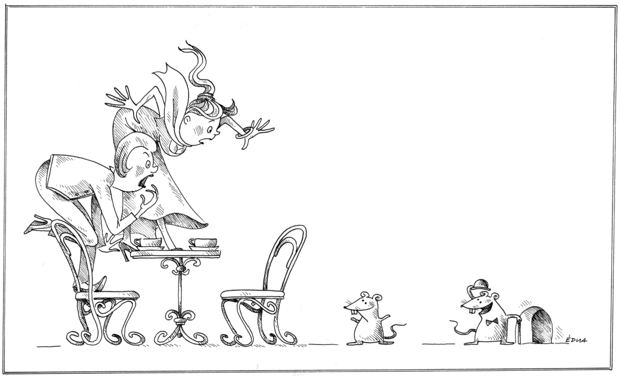

Fig. 3.1. The understanding of signals depends on the structure of the receiving network

shows our anthropocentric view of the world around us. What is 'good' or 'purposeful is called a signal, and what is disturbing, undesirable, residual, is called noise.

However, there may be a better way to discriminate between signal and noise. Perturbations which often reach the network and are large enough to disturb the network structure completely, unless the network develops a specific response, provoke a 'learned, adaptive response'. The same adaptive response may arise after perturbations which bring information about important conditions like food or danger, which were experienced by the network in the past to affect the network's integrity in the long run.[2] These perturbations often lead to an unexpectedly large response, which is not proportional to the magnitude of the perturbation, but has been built in to the structure of the responding network as an adaptive response. We usually call these perturbations signals. Other perturbations which are too small, irregular or unimportant to stimulate a learned, adaptive response of the network, are called noise. *"I get it. When a nice girl enters the classroom, she is a signal for my network. When the geography teacher comes in, he is noise."* Spite, I will show later that a better understanding towards the geography teacher is a key point for the stability of your social network. However, your example shows that the understanding of signals depends on the structure of the receiving network. This is the reason why I will neglect the myriad of network-dependent signals here, and I will restrict my-

[2]Preservation of network integrity – or more properly the giant component of the network, network percolation – is necessary for the survival of the organism built by the network. I will describe this network resilience in detail in Sect. 4.3.

self to a description of the perturbation, which is characteristic of all networks: noise. Resistance to noise is an important feature of network stability.

Intrinsic and extrinsic noise. Various parts of the network, like modules, motifs or elements may cause noise to another network segment. This is called intrinsic noise as opposed to extrinsic noise, which comes from the environment of the network (Swain et al., 2002). In the Internet and the microchip, intrinsic noise can be 100 times greater than extrinsic noise. In contrast, in many other systems the levels of the two types of noise are rather similar (Argollo de Menezes and Barabasi, 2004). In gene networks the intrinsic noise of gene expression comes from the low copy number of messenger RNA. An additional component of noise is the transmitted noise from upstream genes and the global, extrinsic noise, which is correlated with the transmitted noise. Here, the total noise of a gene transcription set was found to be dominated by extrinsic fluctuations, and was therefore a function of network interactions (Pedraza and van Oudenaarden, 2005).

Noises are colorful animals. We have white, pink and brown noise to name but a few. White noise is related to fully random fluctuations showing no correlation in time. Brown noise got the name from Brownian motion, since it is typically present in diffusion processes showing no correlation between increments. In contrast, pink noise (which is also called flicker noise, crackling noise, or in a special case $1/f$, $1/t$ or $1/\tau$ noise) has a memory of past events on all time scales, i.e., it is correlated. $1/f$ noise obeys scale-free statistics, meaning that an event that is an order of magnitude larger may always have a non-zero probability of occurrence, but that this probability is exactly an order of magnitude smaller. For a more detailed description of the various types of noise, see the remark below.

A short course on noise. What is the difference between white, pink and brown noise? Scale-freeness is back again. To understand, how noise is related to scale-freeness, we have to do some mathematics again. Noise is usually characterized by a mathematical trick. The seemingly random fluctuation of the signal is regarded as a sum of sinusoidal waves. The components of the million waves giving the final noise structure are characterized by their frequency. To describe noise, we plot the contribution (called spectral density) of the various waves we use to model the noise as a function of their frequency. This transformation is called a Fourier transformation,

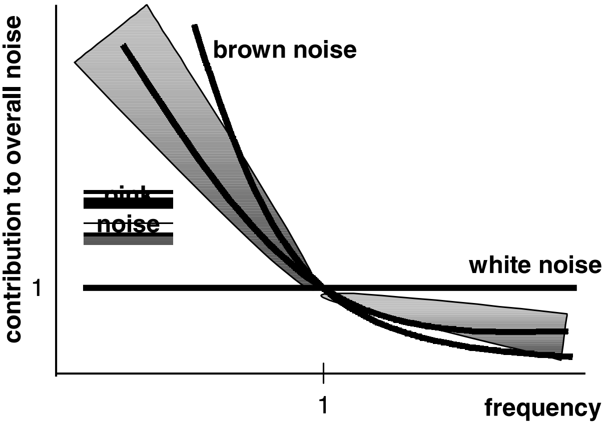

Fig. 3.2. The noise spectrum. Various types of noise (white, brown and pink) are characterized as a sum of sinusoidal waves. This schematic graph shows the contribution of these waves as a function of their frequency. *Shaded areas* around the curve for pink noise are intended to illustrate that pink noise denotes a wide range of noise distributions

and defined by the integral

$$f(\omega) = \int_{-\infty}^{+\infty} f(t)\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}t = \int_{-\infty}^{+\infty} f(t)\Big[\cos(\omega t) - \mathrm{i}\sin(\omega t)\Big]\mathrm{d}t\ ,$$

where ω is the frequency and t the time. At the end of this transformation, we receive a distribution of the constituent waves (see Fig. 3.2). Why do we have this complicated mathematics? Part of it is tradition. Noise was first extensively studied in electric amplifiers, and spectral density and frequency were therefore easy choices, since they were the most important properties characterizing the thermionic tube amplifiers (Johnson, 1925). However, this rather complicated representation has proved to be very useful to discriminate between various types of noise.

- **Scale-free behavior.** The contribution of the sinusoidal waves used to compose the noise (the spectral density of the noise) displays the very same scale-free behavior we already observed in the distribution of various network features.[3] The spectral density of the noise obeys the equation $P = cD^{-\alpha}$. Note that this is the same equation we encountered in Sect. 2.2, where P is now the spectral density of the noise, c a constant, D the frequency, and α a scaling exponent. The values of α can vary between zero and (usually) two. If α is zero, we talk about white noise; if

[3] Just as a reminder, our current scale-free list consists of degree distribution, fractal behavior, event probability, and link-weight distribution (see Sects. 2.2 and 2.4).

α is two, we have brown noise; and anything in-between corresponds to pink noise. In some cases we have noise with α larger than two. This is called black noise.

- **White noise.** White noise (when α is zero) has an equal contribution from each wave throughout the whole spectrum. In other words, white noise implies that the value has fully random fluctuations with no correlation in time. White noise has no memory, e.g., it is a Markov process.[4] Compared to other types of noise, white noise shows a strong dependence on short time scale events. This means that high frequencies – short time scales – contribute equally to the final noise structure with short frequencies. This is not true for any other type of noise. There, short time scales give smaller and smaller contributions as we go from pink noise to brown and black noise.
- **Brown noise.** Brown noise (when α is two) is also called Brownian noise, since it resembles a diffusion process with no correlation between increments. However, brown noise is 'better' than white noise, since it 'remembers' the position immediately before the last time step. Here the starting point is defined, but the endpoint of the given step is random. Compared to other types of noise, brown noise shows a strong dependence on long time scale events. This means that small frequencies – long time scales – give a much greater contribution to the final noise structure than in either white or pink noise.
- **Brown and white noise are related.** Brownian motion can be considered as an integral of a white noise process. In other words, if a particle undergoes diffusion (Brownian motion), its position has brown noise, while its velocity shows white noise, in agreement with the above notion that the next position is selected in a random process.
- **Pink noise.** Pink noise (when α lies between zero and two) is the most exciting of all, and therefore, has many other names. It is also called colored noise, flicker noise, crackling noise, Barkhausen noise, $1/f$, $1/t$ or $1/\tau$ noise. The latter three names refer to the situation when α is exactly unity, and the spectral intensity is inversely proportional to the frequency in the equation above. In pink noise, the contribution of low-frequency waves is higher than in white noise. This means that rare events have a greater effect on the noise than frequent events. This is the reason why we call this noise pink. Its spectrum is biased towards the low frequencies, which correspond to red light in the spectral analogy with visible light. The spectrum of pink noise is therefore 'reddened' compared to white noise, i.e., it is pink. Pink noise contains disturbances equally on all time scales, i.e., pink noise is scale-free. In other words, if we speak about a pink-noise process, fluctuations happening once a minute and once a century have the same influence on the present. Pink noise has a memory

[4] In a Markov process, the distribution of future states depends only on the present state and not on how it arrived in the present state.

of past events on all time scales (Halley, 1996; Milotti, 2002; Sethna et al., 2001).

Pink noise is encountered in a wide variety of systems, such as quasar emissions, solar flares, protein dynamics, human cognition, electronic devices, traffic flow, group decision-making, and economics to name but a few, and is suggested to be a characteristic feature of system complexity (Gilden et al, 1995; Gisiger, 2001; Halley, 1996; Lu and Hamilton, 1991; Milotti, 2002; Sethna et al., 2001). However, there are examples of pink noise closer to everyday life. The crackling noise we hear when we crumple a piece of paper also has the structure of pink noise. Additional examples of pink noise will be listed when we consider a specific case of its occurrence, viz., self-organized criticality, in the next section (Bak et al., 1987; Bak, 1996). Let me note here, however, that some of my former examples of scale-freeness in Sect. 2.2, like music, were also various forms of pink noise.

Noise is bad for the network, if high and continuous noise levels disturb all network functions. So far, the take-home message is that we have to stop noise in order to survive. This assumption is wrong. Reducing the noise to zero would mean no interaction of the network with the environment. Isolation is clearly a bad strategy, since such an isolated network will die. However, zero noise is bad for another reason too. Noise can be helpful in many ways. The first documented observations of good noise were sailors' reports on the peculiar phenomenon that disordered raindrops falling on the ocean can calm rough seas (Reynolds, 1900). Another example of the optimal level of noise is opinion formation. A low noise is not enough for modulation of opinion formation, while strong fluctuations prevent the formation of a definitive collective opinion (Kuperman and Zanette, 2001).

A special case of good noise is stochastic resonance (Benzi et al., 1981; Paulsson et al., 2000).[5] Stochastic resonance occurs when the signal-to-noise ratio of a nonlinear device is maximized for a moderate value of noise intensity. The term 'resonance' in the expression 'stochastic resonance' reflects the fact that the weak signal is often a periodic signal. Moreover, a bistable system can be treated as an oscillator, where the rate of switching events gives the typical frequency,

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[5] Stochastic resonance has a conceptual resemblance with stochastic focusing. In the case of stochastic focusing, the 'helpful' noise is mostly intrinsic, while in the case of stochastic resonance, it comes mostly from the environment (Paulsson et al., 2000). Since intrinsic and extrinsic noise are often difficult to discriminate, I will use the term stochastic resonance to describe both phenomena throughout the text.

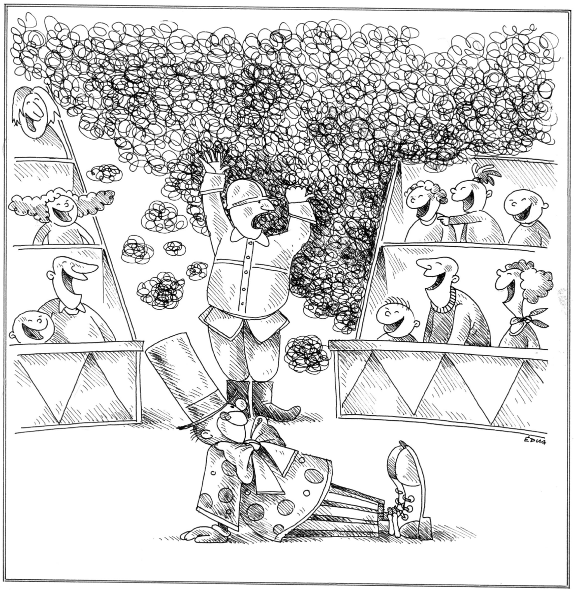

Fig. 3.3. The signal usually has to exceed a threshold to trigger an effect from the adapted network

or eigenfrequency, of the system. This periodic or quasi-periodic signal can be in resonance with the noise, when the eigenfrequency of the nonlinear oscillator and the frequency of the input noise match. A typical case of stochastic resonance occurs in biological systems, when the noise is added to a subthreshold signal and brings it above the threshold, i.e., makes it detectable (see Fig. 3.4). Here, for low noise, the signal will not pass the threshold, and the signal-to-noise ratio is therefore low, due to the undetectable signal. For large noise, the output is dominated by the noise, which leads to a low signal-to-noise ratio again. For moderate intensities, the noise allows the signal to reach threshold, but the noise intensity is not so large as to dominate the output. Hence, a plot of signal-to-noise ratio as a function of noise intensity has a maximum. Pink noise is especially good at helping signals to exceed a threshold, since it is long-range correlated and has a greater chance of satisfying the resonance condition above (Soma et al, 2003).

Stochastic resonance has been invoked to explain climate fluctuations, the sensitivity of fish, cricket, rat mechanoreceptor cells, and the sensitive functioning of ion channels (Bezrukov and Vodyanoy, 1995; Ganopolski and Rahmstorf, 2002; Paulsson et al., 2000; Wiesenfeld and

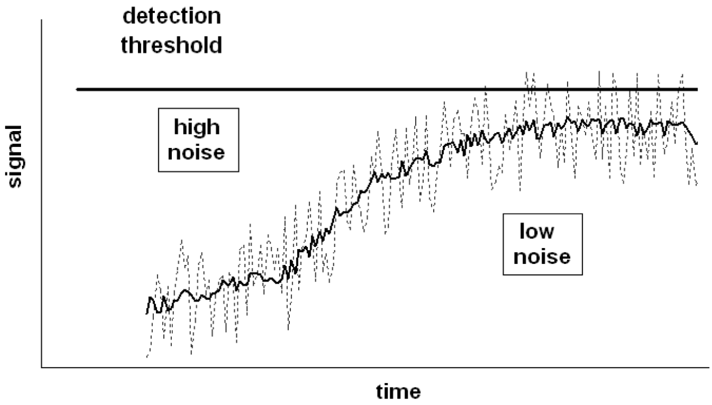

Fig. 3.4. The most important good noise: stochastic resonance. Stochastic resonance is the phenomenon in which a weak signal occasionally exceeds the otherwise limiting detection threshold with the help of noise. In this highly schematic illustration, an arbitrary example of this phenomenon is shown

Jaramillo, 1998). Without stochastic resonance, we would not hear well and most probably would not smell or see well either. Fish find more food (Russell et al., 1999), bones grow faster (Tanaka et al., 2003), and even memory retrieval is better (Usher and Feingold, 2000) in the presence of the appropriate noise. We may conclude that noise can be good, since noise is needed for all the pleasures of life.

If you want to learn, listen to Mozart, not Schoenberg. Memory retrieval has been shown to be higher in the presence of noise (Usher and Feingold, 2000). Various types of music from medieval songs through Mozart and the Beatles all have pink noise structure (Hsu and Hsu, 1991; Voss and Clarke, 1975). The combination of these two observations may explain why many people can learn better if there is music in the background. But does music always help us to learn? Pink noise is efficient at inducing stochastic resonance (Soma et al., 2003) which may help learning better than other noise structures. Schoenberg's music, and some other types of modern music do not have a pink noise structure (Hsu and Hsu, 1991; Voss and Clarke, 1975). Moreover, as the difficulty of a task increases, noise does not help, but disturbs (Usher and Feingold, 2000). This reminds me, to my great surprise, how my favorite Mozart pieces, which had helped me through all my chemistry exams, suddenly became a distracting disaster when I started to learn about Hilbert spaces in mathematics.

Optimal noise not only helps stochastic resonance, but also develops diversity in various networks. If identical networks have stochastic resonance, the time and probability when they reach the threshold and start to behave differently will not be the same. Moreover, noise amplifies selection in finite populations, increases fitness and may increase the chances of robust systems for evolution (Krakauer and Sasaki, 2002).

More reasons for needing noise. The housekeeping heat of steady-state thermodynamics. Oono and Paniconi (1998) suggested a very helpful thermodynamic description for the steady states of non-equilibrium systems. A testable prediction of their theory was suggested (Hatano and Sasa, 2001), and verified later (Trepagnier et al., 2004). A special feature of steady-state thermodynamics is housekeeping heat. This housekeeping heat is defined as the energy preventing non-equilibrium steady states from shifting to equilibrium. Self-organizing networks suffer various types of random damage. Therefore, if the network remained static, it would soon become dysfunctional. Some networks have developed highly specific screening systems which recognize and repair random damage. On the one hand, this process requires energy, which arrives in the form of perturbations or noise. On the other hand, noise-triggered network restructuring will repeat a few steps of the original self-organization and therefore constitutes a much cheaper way of providing a continuous repair function, with the additional advantage that it is always adaptive with respect to the actual environment of the network. The need for permanent noise for the continuous restructuring of networks resembles the housekeeping heat in the steady-state thermodynamics of Oono and Paniconi (1998).

In conclusion, optimal noise is an important condition for network stability, fitness and survival. To achieve optimal noise, a well-functioning, stable network should develop noise control. Networks have figured out many tricks for this. Special arrangements of a few network elements, called network motifs (Milo et al., 2002), may efficiently decrease noise. An excellent demonstration of this phenomenon was given by Becskei and Serrano in 2000, when they showed that, if they put negative feedback into a transcriptional system, the transcriptional noise would decrease. Network modules also protect against noise, since they restrict noise propagation. This happens via the weak links which connect modules and often break if an unusually large perturbation arrives. In the next three sections, I will show what happens in a network when noise arrives.

3.2 Life as a Relaxation Phenomenon: Dissipate Locally, Connect Globally

When the original status of a network suffers a perturbation, the network usually dissipates the disturbing effect, which means that the change is distributed over various elements of the network and relaxation occurs: the network returns to equilibrium conditions. To get distributed, the perturbation has to propagate through the links of the network. Here we have two basic scenarios. The first is when the perturbation can travel undisturbed, and the other, when the perturbation gets stuck at a given node. What are the chances of a perturbation getting around undisturbed? To find the answer, we must first describe the playground for this game. What are the possible paths that the perturbation may select for its round trip? I will discuss the various options, and whether this round trip goes smoothly or the perturbation is arrested at a given point. I will also explain the need for a dual action in the connectivity of networks. On the one hand, free travel of the perturbation has to be confined to a segment of the network causing a local dissipation. On the other hand, the reason for the existence of networks is a global communication spanning the entire network.[6] Finally, I will describe what happens if the perturbation piles up at the origin or any other point of the network. In fact, the following two sections will also elaborate on this latter problem, showing how perturbation-induced damage gets magnified as conditions worsen.

3.2.1 Confined Relaxation with Global Connection

Bearing in mind the above arguments which show that perturbations are necessary for the survival of networks, we shall examine how networks can survive the excessive damage they may cause from time to time.

Scene One

To examine how perturbations might get dissipated, we must first explore all possibilities as to where they may go. One of the most important properties of the network is the presence of a giant component. If a network has a giant component, than most of its elements are connected with each other. This property makes the network a network.

[6] I am grateful for the instructive title of the paper by Tewari and Toner (2005), which helped me to generalize this concept.

When the giant component is dissolved, the network becomes a set of isolated subgraphs which are not connected to each other and therefore cannot communicate. If the network was a living entity like a cell, animal, plant or human being, and if its giant component disappeared, the network would die.[7] *"There is something I do not understand here. Do you get rid of your favorite networks like this? They can die, period. When do they die? How can we protect them? I am a network! I want to know how I can survive!"* Wow, Spite, you got quite emotional! Do not worry. The next two sections will give you many examples of how to save your network.

Let me deal here with the other end. How is the giant component born? The answer is: abruptly. As the number of links increases in a network, we eventually reach the percolation threshold and the giant component appears suddenly.[8] Once there is a giant component, perturbations may start their trip. And now comes the really important question: How far can they go?

Scene Two

For the answer to the above question, we will examine the propagation of information in the network: rumors, infections, Pity's love letters during your history class, Spite (*"Wow, this is creepy. How does he know about this? Can he read my thoughts?"*), etc. The perturbation has to be dissipated fast. We therefore require efficient connectivity in the segment of the network that was hit by the perturbation. In other words, we need small-worldness. Lattice networks will have a hard time if a perturbation arrives. As an example, diamond is very hard. However, if it gets overwhelmed by perturbations, it never bends. It breaks. The diamond lattice has difficulty dispersing the trouble.

Scale-freeness also comes in again. In scale-free networks the perturbation can propagate easily, even when the probability of transmission is extremely small (May and Lloyd, 2001). However, in finite scale-free networks, there is a threshold (Dorogovtsev and Mendes, 2002; Park et al., 2005b), which means that below a critical connection (at low transmission probabilities), the perturbation may only have a restricted trip and may even get stuck somewhere in the network. The

[7] Complex network functions – all the characteristic features of a living organism – cannot be performed in the absence of widespread network communication. Life requires the integrity of the networks which form the living organism.

[8] The sudden appearance of the giant component and the emergent properties of the network may have formed the empirical background of the Taoist and later Hegelian dialectic concept of "quantitative changes becoming a qualitative change" (Hegel, 1989).

small worlds of scale-free networks seem to be small in a dual sense. They give long-range connections and they concentrate most of the important connections in a local environment (Lai et al., 2005).

"Your scale-free networks seem to lose out this time, Peter!" Not yet, Spite. We have not yet taken into account connection weights. The chances of the perturbation getting jammed in double scale-free networks, where both the degree distribution and the weight distribution exhibit scale-free behavior, are much smaller than in random networks (Toroczkai and Bassler, 2004). In other words, natural, weighted scale-free networks seem to be quite good at combining fast relaxation and restricted travel of the perturbation. In other words, scale-free networks are rather stable. This explains the earlier findings and assumptions that scale-free topology is an important element in the stabilization of many networks (Fox and Hill, 2001; Barabasi, 2003).

The positive side of traffic jams: spam protection and local dissipation. The relative difficulty in searching scale-free networks increases even further if the network has a modular or hierarchical structure (Rosvall et al., 2005). Disassortative networks, where hubs are only seldom linked to each other, also restrict travel (Brede and Sinha, 2005). These difficulties confine damage due to perturbations and may also help networks to protect local areas from non-related communication (Rosvall et al., 2005). A carefully balanced combination of efficient local dissipation and global communication might be a key point in network design and survival.

What is the difference between the news and perturbations? *"Peter, a question has been repeatedly popping up in my mind for some time now. You said that we need a careful balance between local dissipation and global communication in the network. How does the network know which change is a perturbation that has to be confined and which is a piece of information that has to be transmitted?"* Spite, you have hit upon a good point. I am afraid the exact answer is not really known. I suspect the answer is similar to the one I gave as the difference between a signal and noise. The network considers as information only those perturbations which arrive often enough for it to develop a learned response, or which are important enough for the survival of the network. These perturbations get the network highways to go right round. All other perturbations are brought right round in local segments of a well-designed network, until they level off.

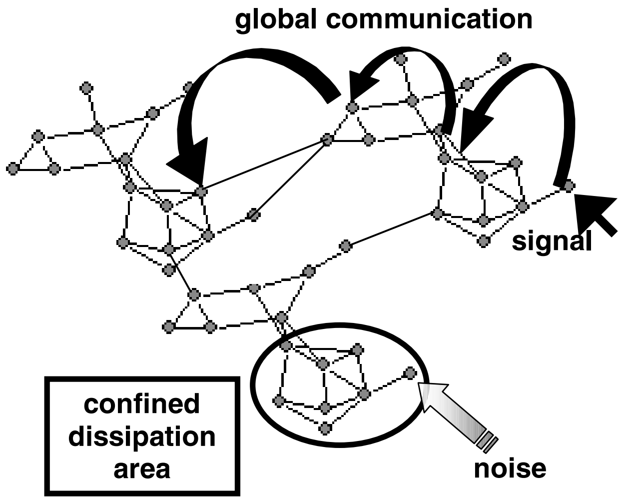

Fig. 3.5. Local dissipation and global communication. Networks should combine the benefits of local dissipation and global communication. Confined dissipation keeps the unspecific perturbation, called noise, in a restricted segment of the network. Global communication helps a few specific perturbations, called signals, for which the network has developed an adaptive response. These signals may reach distant elements of the network

Carefully confined relaxation may be an important element in the way the emerging complexity of self-organizing networks 'develops' stability (see Fig. 3.5). I will discuss an example of this in Chap. 6 in the context of complex gene networks, where a channeling behavior occurs without any extra additional mechanisms (de Visser et al., 2003). Self-organization often leads to the development of scale-freeness, modularity and network hierarchy, causing an 'automatic' stabilization of the network.

3.2.2 Self-Organised Criticality

We have seen that scale-freeness helps the local dissipation of perturbations. But what happens if a perturbation cannot be dissipated and gets stuck? Tension will develop. A very spectacular example of this scenario occurs when the perturbation is continuously repeated and the tension keeps on increasing. The development of tension does not lead to major problems for quite a while, since the individual perturbations arrive separately and get stuck at different points of the network. However, as more and more perturbations arrive, local tensions accumulate and may develop to a point when a propagating relaxation suddenly occurs like a kind of avalanche.

This behavior was called self-organized criticality by Per Bak (Bak et al., 1987; Bak and Paczuski, 1995; Bak, 1996). Unfortunately, self-organized criticality remained a rather loosely defined concept, describing a phenomenon in which, in a network with restricted relaxation, a gradual increase in tension is followed by sudden avalanches.[9] However, self-organized criticality is a very general phenomenon, and it is also very helpful for understanding network behavior. I will describe its meaning and give several examples of similar behavior in the following. At the critical event, a sudden relaxation develops, the effect of the perturbation starts to propagate, and finally a larger segment of the network will communicate. A netquake occurs. The extent of netquakes has a scale-free distribution in the given network, both in spatial extent and duration (Bak et al., 1987; Bak and Paczuski, 1995; Bak, 1996).[10] The frequency of occurrence also has a scale-free distribution. This shows that there is a correlation between netquakes, i.e., the netquake frequency depends on the whole history of the system (Lippiello et al., 2005). This is in fact true of most natural, self-organizing, critical events, and reflects the general behavior of pink-noise structures as described above (details will be given later).

Self organized criticality may lead to the development of scale-free networks. Self organized criticality reorganizes the network structure as the avalanches pass away. Elements will be disconnected, then reconnected again. Fronczak et al. (2005) showed that the network reorganization may find the equilibrium in which both the degrees and the avalanche properties exhibit a scale-free distribution.

[9] Originally, Per Bak and coworkers (1996) defined self-organized criticality for a rather well-characterized set of events involving sand piles, or rice piles. In this chapter, I will considerably generalize this concept to demonstrate its applicability in our everyday life. This may be misleading. I would like to ask the patience of those who prefer exact definitions and better defined concepts.

[10] Netquakes may be regarded as events in which the barriers confining perturbations to certain network areas start to break, and as perturbations start to communicate, more and more barriers cease to exist, or are overcome. Perturbations fluctuate continuously and get dissipated to different extents at a given instant of time. Consequently, a netquake is a rather stochastic process which soon stops in most cases. However, some exceptional netquakes propagate violently. The successive completion of distinct events during the occurrence of a netquake may explain the scale-free behavior of netquakes (see Sect. 2.2).

Netquakes are violations of the fluctuation–dissipation theorem. The fluctuation–dissipation theorem is a famous statement of statistical physics related to the phenomenon that a fluctuation in a system is dissipated as it returns towards equilibrium. A well-known form of this statement is the Stokes–Einstein relation between diffusion and viscosity, stating that $D = T/c\eta$, where D is the diffusion constant, T is the temperature measured in kelvins, c is a constant, and η is the viscosity. Here the fluctuation is Brownian motion, dissipated by resistance due to the viscosity of the surrounding material. Heterogeneous and/or slowly relaxing, glass-like phases can avoid equilibrium in the long term, thus violating the fluctuation–dissipation theorem. Here again, relaxation is confined and happens by directing the perturbation to the fastest relaxing segment of the network (Grigera and Israeloff, 1999). The extent of the violation of the fluctuation–dissipation theorem follows a scale-free distribution (Bonn and Kegel, 2003). This is in agreement with the scale-free behavior of netquakes, which probably occur in most cases here. In fact, the deviation from the fluctuation–dissipation theorem can be used as a measure of metastability (Bonn and Kegel, 2003), and maybe also as a measure of system complexity (the definition of complexity will be detailed in Sect. 4.4).

Netquakes can have a wide variety of forms. Earthquakes, landslides, forest fires, fractures, volcanic eruptions, avalanches, protein quakes, magnetization propagation (the Barkhauser effect), quasar emissions, solar flares, dripping faucets, rain, and many more examples mentioned in the last section, like the crackling noise we hear when we crumple a piece of paper, belong to the class of obstructed relaxation events and all conform to the rules of self-organized criticality (Alessandro et al., 1990; Bak, 1996; Bazant, 2004; Cote and Meisel, 1991; Gilden et al., 1995; Gisiger, 2001; Halley, 1996; Lu and Hamilton, 1991; Malamud et al., 1998; Milotti, 2002; Penna et al., 1995; Sethna et al., 2001; Turcotte, 1999).

Panic quake. One of the recent developments of self-organized phenomena came from Hungary (Helbing et al., 2000) and the Philippines (Saloma et al., 2003). Humans turn into a flock of sheep if a real danger arrives. This herding behavior is known scientifically as an allelomimetic tendency. Our most important decisions are made by our emotions (Damasio, 1994; Rolls, 1999), and this old thalamic system most probably takes over in panic reactions. We trample each other, form arches in front of exits, effectively block the only escape route, do not use alternative escape routes, etc. (Helbing et al., 2000). Saloma et al. (2003) studied mice, not humans. On the basis of the above notes, this would not seem to make a significant difference! There was a difference though. Unlike some of us, mice do not like to swim. The finding was that they left the pool in groups with a scale-free distribution. These mice seem to be as socially handicapped as we are. They probably watched the seniors. Would they dare to jump out? Nothing happened for quite a while, but tension built up. And then, suddenly one mouse equivalent of spiderman jumped and a whole flock followed immediately, causing an instant relaxation, or panic quake.

The nature of thunder and lightning. There are many more phenomena which behave rather similarly but have not yet been classified as networks reaching self-organized criticality. One of these is thunder and lightning. Here we may also discover all the items in the inventory: the series of events helping static to grow, thereby developing tension in the form of a voltage, and the sudden relaxation at the end (Nebuchadnezzar might have a story like this to tell!). It would be nice to establish the scale-free distribution of lightning intensities and the loudness of thunder. In fact, the $1/f$ bursts[11] of very low-frequency electromagnetic radiation coming from lightning discharges (Magnasco, 2000) strongly suggest that this is the case.

Fortunately, volcanic eruptions, landslides, and panic are not common everyday experiences. Protein quakes are too small and solar flares are too distant to observe. (So astronomers say. For the solar flare, you have only two opportunities with a telescope and bare eyes: one for your right eye and another for the remaining left!) However, there is one very important example of self-organized criticality at each moment of our life: lung quakes. Whenever we take a breath, the individual airways open in an avalanche-like fashion, producing the same scale-free statistics as all the other critical phenomena listed above (Barabasi et al., 1996; Suki et al., 1994).[12] A few more common examples of netquakes will be listed here.

Tick quakes. Self-organized criticality may be much more common than we think. Peterson and Leckman (1998) showed that ticks,

[11] This is the same as $1/t$ noise, an example of pink noise (see Sect. 2.1).

[12] Actually, the ‘reverse phenomenon’, coughing, may also be a self-organized critical event and asthmatic bronchoconstriction has recently been described as a self-organized event (Venegas et al., 2005).

which are rapid, brief, unintentional skeletal or vocal muscle movements in some people, like those with a Tourette syndrome, follow scale-free statistics. Ticks may in fact be muscle quakes, exhibiting bursts of the tension which has gradually built up in these patients and could not be gradually dissipated as it would in the rest of us.[13]

Gossip quakes. When a gossiper hears a great story, tension develops: "I should tell someone this!" The more stories arrive, the more likely a gossip quake is to occur. Eventually, the gossiper cannot resist any longer and picks up the phone. It may be worth asking whether time intervals between the transmission of two consecutive pieces of gossip obey scale-free statistics.

In all these events, both the probability and the magnitude of the event follow pink noise (Milotti, 2002; Sethna et al., 2001). What does this mean? If the event has $1/f$ noise statistics, there is a non-zero probability for any event which is an order of magnitude larger, but this probability is an order of magnitude smaller. We have a clear sense for this in the case of rain or earthquakes. Fortunately it is only very seldom that we have a devastating earthquake, or rainfall so heavy that it smashes everything or washes everything away. However, this is a general phenomenon. So watch out next time you start to crumple your candy paper at the movies. If you are truly unlucky, it may make a really big noise. *"I cannot believe that I will ever go deaf after crumpling a candy paper at the movies."* Yes, Spite, you are right. Natural scale-free events usually have an exponential cutoff. So there is no need to worry about going deaf due to a sudden noise explosion from your candy paper.

Our life is full of netquakes, from birth to death. Let me list a few examples here:

Ogling quakes. A girl and a boy travel on the bus. After the first glance, they both realize that the other is the most beautiful, most charming person they ever saw. Good manners, however, require them to stop gazing at each other after a second or so. "I need to see her again!" As the seconds pass, a significant tension develops. All of a sudden, good manners are forgotten, and they start ogling again. What is the distribution

[13] If you are a psychologist and after this sentence you want to throw the whole book into the fire, I ask you to consider the fact that this response might reveal a tension–relaxation problem.

of the length of gaze-abstaining periods? What is the distribution of gazing intervals? It is probably scale-free.

Woo quakes. Suppose the girl and the boy finally met. Now it is said that love makes one beautiful. If you have ever watched an amorous girl or boy, you will certainly agree that the statement is true. But why do we look more beautiful when we are in love? We do not look different. We start to behave differently. We become more playful. What does this mean? Among the many changes, we may observe sudden, unexpected movements and a kind of indolence. What is the driving force behind these movements? If everything goes well, a kind of continuously growing tension develops, pushing us to do things that are not really the right thing to do in the middle of the street, on the subway, etc. What is actually happening? Sooner or later a pseudo-action will follow. The tension relaxes by an unexpected small jump, by a sudden kiss, or a smile to a complete stranger. The relaxation is not complete, however. A large amount of this happy tension remains, and starts to grow again. What do we call these relaxations? They are woo quakes. What is the most probable distribution of the periodicity and extent of unexpected, playful actions? It is scale-free.

Sex quakes. Let me continue the above story. Imagine that the same couple finally reach a comfortable apartment, or a less comfortable glade in the forest, a boat, a space station, a live show, wherever their habits and possibilities may have brought them. Continuous – or perhaps it is better to say rhythmical – input of energy, gradual growth of tension and then, suddenly: BANG! The relaxation occurs like an avalanche, like the eruption of a volcano. Is this familiar to you? Evolution seems to figure out a self-imposed method for rehearsing scale-freeness in a joyful manner. Sometimes it is good, sometimes it is better. Can it be even better? If this is really scale-free, our happy couple always has a chance for an order of magnitude larger effect, although the chances of this are an order of magnitude lower. The scale here is probably rather limited. Are you sure? Our couple is like the inhabitants of Los Angeles. They can always hope that the next will be the Really Big One. I must admit, my example is perhaps not the best this time, since the emotional background of the two expectations seems to differ. *"Peter, let me remind you of what you said just a few lines above: natural scale-free events usually have an exponential cutoff. It might be disappointing to you but the same principle has to be applied here, I am afraid."*

Baby quakes. It is now nine months later. I see a cute baby with a comforter. Babies may use their comforters to relax a multitude of tensions which reach them continuously from the new and alien world. In the absence of comforter-induced relaxation, crying (a baby quake) develops. Both suckling of the comforter and crying may follow a scale-free distribution, although obviously in periods when the baby is awake.

Growth quakes. Let us take one more step. Our baby starts to grow. The young cells keep dividing. However, divisions themselves are multiply regulated and the growth of different tissues is not synchronized. A tension develops and growth becomes saltatoric (Lampl et al., 1992). Uneven growth brings us beyond self-organized criticality, since here tension development is not steady but follows a multidimensional pattern. Instead of exhibiting a simple scale-freeness, growth quakes may resemble the multifractal properties of heartbeats which are described in Sect. 7.2. Indeed, later studies (Thalange et al., 1996) showed that the rules for saltatoric growth can be very complex. Uneven growth is used by plants to change their shape or catapult their seeds, and has become an important category in evolutionary economy (Feenstra, 1996). I will spare you from a quake version of 'Buddenbrook House' to describe the life of the happy couple, but I hope I have convinced you that, until the very end (crying quakes of the same baby, now a little older, at their funeral), their life will be full of similar events.

In the self-organized critical state, weak links break first (behaving like the weakest link of the chain). In the netquake, the rearrangement of weak links helps to restabilize the system. In other words (Sethna et al., 2001): "Not all systems crackle. Some respond to external forces with many similar-sized, small events (for example popcorn popping as it is heated). Others give way in one single event (for example, chalk snapping as it is pressed). In broad terms, crackling noise is in between these limits: when the connections between parts of the system are stronger than in popcorn, but weaker than in the grains making up chalk, the yielding events can span many size scales. Crackling forms the transition between snapping and popping." Crackling needs weak links.

Culture quakes. When I wrote the first version of this book, we had the 40th anniversary of the Beatles avalanche in the USA in 1964. Why did they reach such a large segment of the society in such an incredibly short time? Why do cultural and technological innovations resemble self-organized criticality? Innovations should reach a percolation threshold (Ryan and Gross, 1943), after which they suddenly became a general custom of the majority of the social network. The acceptance of innovations is a cooperative action, which inherently contains the possibility of avalanches (Watts, 2002) and has indeed been shown to behave as a self-organized critical phenomenon for the change in pottery styles and for the propagation of the idea of self-organized criticality itself (Bentley and Maschner, 2000; 2001). Weak links may also contribute to the occasional burst of innovations, novel ideas or cultural changes to the whole society. These innovations often come from a module of the society[14] which is connected to the rest by weak links. Once the innovation has passed this bottleneck, as the Beatles crossed the ocean, it gets a free ticket to ride around.

Schumpeterian innovation clusters. The phenomenon of innovations breaking away from a state of relative isolation – combined with the notions of cooperation and the percolation threshold – may also help to explain Schumpeterian clustering of innovations (Schumpeter, 1947), i.e., a surprising onset of serial innovations behaving like bursts, and coming in clusters. As innovations spread in society, they will sooner or later reach someone who may add a new element to the existing design. The number of persons reached by the innovation probably follows scale-free statistics. Moreover, the innovative process itself consists of individual steps, where the next step depends on the one before. These processes also display a scale-free pattern.

To sum up, if we seek network stability (and so we should, otherwise we die),indexdeath we need a highly efficient but local relaxation. To achieve this, the perturbation has to reach the elements of the given segment of the network rapidly enough. In other words, we need small-worldness for efficient local dissipation. We also need small-worldness for the global communication of signals. On the other hand, we need scale-freeness and nestedness (in the form of network modularity) to confine the perturbations. Efficient networks combine global connections with local relaxation.

If many perturbations arrive and get arrested one by one, avalanches will occur sooner or later. For this latter event, we need a constant inflow of perturbations, the subsequent development of a tension, and then its sudden relaxation through an avalanche. There is another

[14] I am grateful to Viktor Gaál for this idea.

Fig. 3.6. Crackling forms the transition between snapping and popping

important message here, if we put the last sentence in a slightly different form: for self-organized criticality, we need the constant inflow of energy, the subsequent development of tension, and then its sudden relaxation by an avalanche. For this exposes a surprisingly general meaning. Without a constant inflow of energy, there is no life. The arrival of energy drives the system forward, and by making the system thermodynamically open, allows a decrease in its entropy.[15] However, each energy packet is a new perturbation, and they must all be dissipated. Therefore, relaxation is necessary for life. And we had better ensure that it is fast and confined. Life behaves as a carefully confined relaxation phenomenon in globally connected networks.

[15]The free energy has to decrease for any change in the self-organizing networks. Since the free energy is defined as $G = H - TS$, where G is the Gibbs free energy, H is the energy, T is the temperature measured in kelvins, and S is the entropy, we need an inflow of energy (negative ΔH) to compensate the decrease in entropy (negative ΔS) and keep the free energy change negative.

3.3 Network Failures

Spite! You have become suspiciously silent. It is time to snap out of this dream-world relaxation. Wake up! Here is a question: What happens, if perturbations are not only temporarily, but hopelessly stuck in the network? I see your thoughts are still far away. I will give you the answer: if relaxation becomes impossible, than one or more elements of the network will receive a perturbation big enough to cause the disassembly of the bottom network below the unlucky element where the perturbation got stuck.

Let us consider an example. On 10 August 1996, the failure of a power line in Oregon led to a black-out in several states of the USA and Canada. What happened? Due to a combination of extremely high temperatures and a full power load at about 3:40 pm, the 500 kV line connecting the Keeler and Alston substations in Oregon sagged so much that it touched a tree and was immediately shorted. The resulting frequency oscillations knocked out 13 hydroelectric units at the McNary Dam. The disassembly of these bottom networks cut the main inter-tie, which runs between Oregon and Southern California, causing a chain of breakdown events as far as Nevada, New Mexico and Arizona. This triggered further outages throughout the whole western region of the USA leaving more than 4 million people without electricity in 11 states (O'Donnell, 2003). The national disaster on 10 August started due to relatively minor initial damage. However, this was enough to switch off the 13 hydroelectric units nearby and was immediately magnified. Such behavior is called cascading failure and is typical of network structures. Similar cascading failures have been observed in the economy, in earthquake aftershocks, and in other contexts (Bak, 1996; Moreno et al., 2002). One of the best examples of this is the domino effect. With only a slight touch, the long line of dominos all topple over in a fraction of a minute.

Power net quake. *"This cascading failure reminds me of the disturbing variety of quakes in the previous section. Don't you think you have forgotten to mention something?"* Well done, Spite! You learned your netquake stories very well! Indeed, the cascading power failures can be regarded as events of self-organized criticality in the power net (Carreras et al., 2004). The continuously increasing tension is the increasing load on the system. The continuous input of energy is rather obvious. Relaxation comes in the form of system failure. (Well, this relaxation does not seem to be valid for the controllers!)

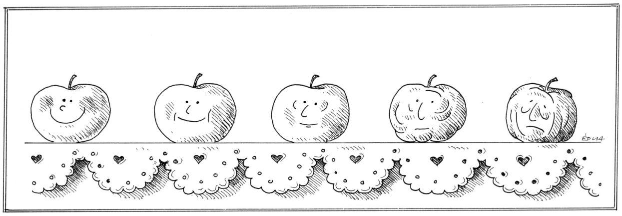

Fig. 3.7. Permanent random damage can lead to a massive network malfunction

This is bad enough. If we cannot design systems wisely, to save our network from cascading damage, we may start to collect ice cubes on every hot day during the summer (or move to Alaska). But the situation is even worse. We do not even need a cascading failure to impair a network. Although scale-free systems are rather insensitive to random damage (which is why evolution did not throw them straight into the garbage can), they are much more sensitive if any of their highly connected nodes, the hubs, gets damaged (Albert et al., 2002). This is what pushed the situation from bad to worse and from worse to disastrous in August 1996, as the local damage disconnected the Oregon–Southern California inter-tie, causing an avalanche of further damage. Cascading failures reach hubs rather quickly, and sooner or later they are sure to reach a hub as they propagate in the network. As seen above, cascading failures may sequentially incapacitate the network up to a critical point where the giant component collapses and the network ceases to function (Moreno et al., 2002).

Error and attack tolerance of scale-free nets. If compared to a random graph, a scale-free network is surprisingly stable against random damage, but vulnerable against a planned attack (Albert et al., 2000; Bollobas, 2001). The following gives a few details concerning error and attack tolerance of various networks:

- **Exception One: When random damage is also bad.** As opposed to occasional random damage, permanent random damage can cause a massive malfunction of the network (Dorogovtsev and Mendes, 2001). In other words, aging can devastate our networks. Not abruptly, but efficiently. Some examples of this are discussed in Sect. 6.4.
- **Exception Two: When a planned attack is not that bad.** Assortativity makes the network resistant against planned attacks. In an assortative network, similarly connected elements associate with each other. Hubs

like hubs, smaller nodes seek nodes, and almost isolated elements will be associated with similarly isolated elements. Such an assortative network is quite typical in social nets. For the most assortative network, ten times as many hubs have to be removed to destroy the giant component than for the most disassortative one (Newman, 2003a). However, assortative networks are more unstable than disassortative ones. In assortative networks, perturbations can propagate further and a better synchronization leads to larger dynamical fluctuations (Brede and Sinha, 2005; di Bernardo et al., 2005).

- **Not all networks are scale-free.** As a final remark, I would like to warn you that it is only because of their wide occurrence and stability that we are talking so much about scale-free networks. There is a whole universe of unexplored network structures out there. To give but one possible example, a simultaneous optimization with respect to both random damage and intentional attack gives novel types of networks (Valente et al., 2004; Paul et al., 2004; Shargel et al., 2003).

Where does the perturbation get stuck? Do perturbations cause random damage? In principle, a perturbation may get stuck anywhere in the network. In practice, this is probably not true. I have not yet found an exact answer to this question. Perturbations may get stuck at narrow points of the network, where the connection is weakest.[16] Alternatively, perturbations may get arrested at hubs, where most traffic converges and causes a jam. In the latter case the perturbation behaves like a terrorist, attacking the most vulnerable points of a scale-free net. Do our networks direct the damage to their most vulnerable points, thereby making us hostages of terrorists? Fortunately not. The giant component of the network is well preserved even in the case of a planned attack. For example, the World Wide Web will remain connected and functional even after the removal of all of its nodes of degree higher than five (Albert and Barabasi, 2002).

The wisdom of our cells: low flux comes with high stability. Enzymes are parts of the metabolic network of our cells. Their stability is governed by the stability of their protein structure. If we introduce a few extra, stabilizing bonds, we will make the mutated protein more rigid. However, increased rigidity leads to diminished enzyme activity (Shoichet et al., 1995). Thus our stable protein will most probably be a 'weak point' of the metabolic net, since it will have a low flux and will offer

[16] These may be weak links between modules, which are often characterized by a high betweenness centrality (Gfeller et al., 2005).

better chances for the perturbation to stop. However, it is not bad if the perturbation stops here, since our 'weak point' is not in fact weak at all: with our stabilizing mutations, it has become more stable than the average protein in the original network. Thus the metabolic network of our cells may contain two types of protein: (1) high achievers, which have a high flux, ensure most of the tasks required for cell survival, but have a low stability and pass the perturbation to their colleagues, and (2) low achievers, which have a low flux, and consequently do not do much work, but have a high stability and absorb the perturbation, thereby saving the high achievers from major damage. If we look around at social networks, we may start to think that this division of labor is rather general in networks.

It is time to turn back to our cascading failures and confirm the initial statement: they are still most annoying. We build up networks at high cost, and yet they sometimes literally melt down in a matter of minutes. We have to make them safer. If the disconnected Oregon–Southern California inter-tie caused such big trouble, why do not we make another, parallel inter-tie? Actually, why do not we duplicate most of these sensitive lines?

Doubling the network seems to be a rather bad idea. According to a very interesting model worked out by Duncan Watts (2002), too many connections may make the extent of damage unpredictable. The Watts model is based on a scenario in which the status of each element of the network depends on the average status of its neighbors. This setup suits the description of any domino-type cascade, like the chain of events in August 1996, where neighbor influence is predominant. If we start to increase the number of connections in this network, we eventually reach a critical point where cascading failures become possible. Here a scale-free distribution of cascade size is observed. This means that most cascades will be small, whereas large, devastating cascades will occur only seldom. Paradoxically, if we then go further, cascades become larger but more seldom. This is due to the dilution effect. When a node has too many neighbors, the influence of each will be diminished. Interestingly, close to this second, critical point, a bimodal size distribution is observed, where large cascades have much greater chances of occurring than in the scale-free distribution regime. This implies a more extreme type of instability which is even harder to predict (Watts, 2002). Thousands of perturbations may arrive without any of them having any effect. And then the proverbial last son of the Perturbation Family gets in, smallest of all, and suddenly, in a single miraculous second, a gigantic cascade erupts. The instability of 'overconnected' networks is rather general and can be applied to food webs, trade networks and power nets (Fink, 1991; May, 1973; Siljak 1978).

Vulnerable points of networks. After the September 11 terrorist attack, a considerable research effort has been directed at identifying the most vulnerable points of our networks and figuring out methods for saving them. It turns out that the original ideas – that hubs are the most sensitive points of scale-free networks (Albert et al, 2000; Jeong et al., 2001) – were only partially true for real, weighted networks. Devastating global cascades are likely to occur if we remove a node with one of the highest loads in the network and if the network previously had a highly heterogeneous load distribution (Lai et al., 2005). Our cells offer an efficient and low-cost experimental field for studying network damage. Node removal here can be achieved by the deletion of a gene from the blueprint of the cell, the DNA, or by destroying the messenger RNA by a method called RNA interference. If the deletion was lethal, we have very likely struck out an important point of the network.[17] Comparison of the lists of lethal genes with their position in cellular networks has revealed that it is difficult to find simple correlations between a given network property and lethality, but a central position in cellular communication was a rather good predictor of the lethality of the deletion (Coulomb et al., 2005; Estrada, 2005; Schmith et al., 2005).

Our original plans for a giant network of parallel inter-ties are thus rejected. Should we give up? Should we live in uncertainty? No, there are ways to solve the problem of cascading failures.

Tricks to protect our networks against cascading failures. So far I have not mentioned the easiest trick: do not make more ties, make bigger ones! Increase the capacity of the whole network. As the capacity is increased, cascades will be diminished (Lee et al., 2005a). However, this requires enormous resources which are not often available. There are other solutions:

- **Redistribute the load.** If the traffic of the most central nodes is redistributed to other, non-central nodes, the network capacity can be increased by a factor of ten (Ghim et al., 2004; Yan et al., 2005).

[17] The interpretation of individual data is rather difficult here, since the deletion of an enzyme protein offering a unique reaction, which produces an essential molecule for cell survival, is certainly lethal, and yet this peculiar protein will never pop up in any network analysis as 'important'.

- **Rewire in the neighborhood of the damage.** If a node is destroyed, an emergency rewiring of its neighbors can save the network (Hayashi and Miyazaki, 2005). We may make these rewired substituting nets around the most vulnerable nodes as a precautionary measure.
- **Damage the network, if you want to save it!** This rather tricky method has been described by Motter (2004), who proved that the selective removal of network elements and links with either a small load or a large excess of overload also diminishes the size of the cascading damage.

I have left the really big invention until last on the list of rescue efforts. This is the modular structure of the networks. In fact, this is what was actually implemented after the power-failure disasters in the USA (O'Donnell, 2003). Cascading failures can be stopped at intermodular borders. *"Wow! Do cascading failures arrive at the bridge connecting the two sides of the border, and start to shout: 'Damn it! I left my passport at home again!' "* No, Spite, the real situation is more striking than that. When cascading failures arrive at the bridge connecting the two sides of the border, the bridge starts to shout: "Damn it! Cascading failures!", and then collapses. The central idea of the book, weak-linkness comes in again here. Intermodular contacts are often formed by weak links. These contacts may behave like a fuse and melt when the damage arrives. *"Peter, your brain seems to have got boiled in the hot August of 1996. You just made a big issue about the break of the Oregon–Southern California inter-tie and now, what do you want to sell us? That your favorites, those fabulous weak links are good because they break."* I know, this is the point when you throw the book into the trash can for the 67th time.[18] The key point is this: after breaking the inter-tie, the two electrical systems were still connected and the damage could still propagate from one to the other. If I can *really* disconnect the module – because it is linked to others *exclusively* by weak links, and not by strong inter-ties, where one may melt but the others still remain as viable connections – the module may become self-sustaining and still be able to function.

Weak links and microcracks. Intermodular weak links help a number of natural networks. As one example, the fracture process in various materials depends strongly on their heterogeneity. Heterogeneous materials produce microcracks which begin to merge to reach a critical density. Here the microphases (modules) are connected by weak links (weaker forces) allowing a

[18] This actually means that my beloved work has already been fished out of the trash 66 times: I thank the reader! If it is getting too dirty, you may just go and buy another one. I promise the publisher will love both of us.

selective relaxation of the tension by the disconnection of the modules. Perfect crystals have no weak links and no modules, so that they break unpredictably, causing a sudden and devastating decomposition of their structure and often producing a scale-free fragment size distribution (Bazant, 2004; Kun et al., 2005; Sornette, 2002).

In conclusion, if you witness a constant inflow of perturbations, prepare for the worst. A cascading failure may start, leading to devastating damage in your network. A scale-free degree distribution will help a lot to fend this danger off. However, perturbations do not cause random damage. They act like terrorists, often preferring hubs and critical connective elements between distant segments of the network. Therefore, modules become critical for stopping a damage avalanche. Weak links help to control cascading damage, and thus save the connectivity and the life of the network.

3.4 Topological Phase Transitions of Networks

We may feel better after the previous section. If we have any trouble with network construction, or a number of perturbations get stuck, we do not necessarily have to find a new network. Our old network can be rescued. But what happens if our network gets completely saturated with perturbations? Should we give up?

Why are more perturbations worse than a few? So far I have been unable to find an elegant proof of this instinctive truth. Networks may have a characteristic relaxation time. If the next perturbation arrives sooner than this, it cannot be dissipated and the new perturbations just pile up.

However many perturbations arrive, a good networker – and let me invite you to join the club – never gives up. There is still one more way to escape: a topological phase transition. Networks may undergo a series of interesting transformations called topological phase transitions. A topological phase transition occurs if the continuous increase in the number of perturbations provokes a singular change in the global topology of the network. The global topology is best monitored by the measure G/N, where G is the size of the largest connected component

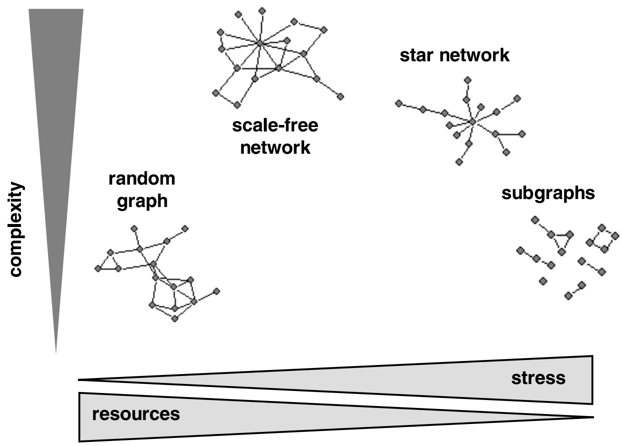

Fig. 3.8. Topological phase transitions of networks. The figure shows the topological phase transitions random graph → scale-free → star phase → disintegrated, fully connected subgraph (Derenyi et al., 2004; Palla et al., 2004) as resources become more and more limited or stress grows. Note that resources and stress have been substituted for the network temperature used in the original publications. The level of complexity, and also the random and scale-free graphs, are merely illustrative

of the network and N is the total number of its links (Derenyi et al., 2004; Palla et al., 2004).[19]

If we keep the network at a high temperature and perturbations arrive continuously, new connections can easily be formed. Under these conditions, the whole network will resemble an Erdős–Rényi-type random graph (Erdős and Rényi, 1959; 1960). In such networks all connections are formed at random, and all elements have the same probability of being connected. If we lower the temperature, a condensation occurs, where compactness is increased and higher degrees are preferred. The network will first develop a scale-free degree distribution. Finally, it will arrive at a star conformation where one or very few mega-hubs dominate the whole system (see Fig. 3.8).[20] The star network resembles dictatorships in social networks. If we decrease the temperature further, the star phase will condense even more, which is possible only

[19] Alternatively, the measure $k_{\max}/M$ can also be used, where $k_{\max}$ is the largest degree of the network and M is the number of edges in the network (Derenyi et al., 2004; Palla et al., 2004).

[20] A similar emergence of the scale-free network together with transitions between graphs with variable degrees, star phase and random structures has been described by Biely and Thurner (2005).

by disintegrating the original network to produce fully connected subgraphs. This means the disappearance of the giant component, and if the network was a living system, this topological phase transition would be called death.

The ultimate peace: a single, fully connected graph. If we decrease the temperature even further and the network approaches an absolute, undisturbed peace, the isolated, fully connected subgraphs will condense to a single, fully connected graph in the physical model (Derenyi et al., 2004). The meaning and significance of this condensation is currently unknown in biological or social networks. Most probably, the fully undisturbed phase is so hypothetical in these systems that we actually never experience the fully connected full graph phase. We can only speculate! If the subgraph phase corresponds to the death of biological networks, what is the phase which comes after it, when things get unrealistically peaceful?

Topological phase transition reflected by the scaling exponent. Let us return to the general formula for the degree distribution in scale-free networks: $P = cD^{-\alpha}$, where P is the probability of the given degree in the network, c is a constant, D is the degree, and α is the scaling exponent. Using the expression above, we may describe the topological phase transitions as a gradual change in the exponent α. The exponent starts from 1 (denoting random networks), grows until about 4 (denoting scale-free networks, see Table 2.1), and then grows further to even higher numbers showing the presence of fewer and fewer hubs with more and more connections. As the scaling exponent α becomes larger, the degree distribution will shift towards an exponential decrease, implying a rapidly decreasing number of highly connected elements and reaching the star phase as an extreme case.

We can transpose the driving force of the above topological phase transitions, the network temperature, to a different context. At high temperatures, our lucky network enjoys a large amount of energy and has a high number of connections. Even if the perturbations cause rather significant damage, there is a good chance that the giant component of the network will still be preserved. Moreover, the high energy contributes to a fast rebuilding of destroyed connections. On the other hand, if the network is in a low-temperature, low-energy environment, the average connectivity is lower. The chances grow that the perturbations will hit vital connections which do not have a backup version in the network. As an additional angle, if we have a large amount of energy around, perturbations arrive almost continuously. They are smooth and predictable. As the outside energy becomes lower, the distribution of perturbations becomes uneven and unpredictable. This is a phenomenon I will call stress in this book. In this context increasing stress provokes topological phase transitions (see Fig. 3.8).

Definitions of stress in physiology and physics. In physiology, the word 'stress' was coined by Hans Selye (1955, 1956), who used the term for a wide range of strong external stimuli, both physiological and psychological, which cause a general protective response, the stress response. In agreement with his definition, from the network standpoint, stress is any large, unexpected, and sudden perturbation of the cellular network, to which the network (1) does not have a prepared adaptive response or (2) does not have time to mobilize its adaptive response. In the latter case, the strong external stimuli overwhelm the elements of the biological networks and fail to provoke the learned, adaptive response. Stress in this book will be used differently from stress in the usual sense in physics, where it is a force that produces strain in a physical body.

Topological phase transitions follow the parsimony principle. Star networks involve a lower cost than either random graphs or scale-free nets, since star networks require less wiring than either of the other two configurations, but still provide a full connectivity of the elements. Bentley and Maschner (2000) showed a random → scale-free transition in the development of a citation network as resources became more sparse, implying that fewer journals wanted to publish the articles on those topics. Similarly, Stark and Vedres (2002) described random → scale-free → star topological phase transitions of business consortiums as the economy worsened. A decrease in connections as a source of stabilization has also been demonstrated with a simpler network model in early work by Gardner and Ashby (1970).

It is now time to go back to our starting point: an overwhelming number of perturbations. When a perturbation gets arrested in the network, the unlucky element at which this happens receives a lot of focused energy. In extreme cases, whatever old connections there were around this element stop functioning and the element is free to seek out and build up new connections. In principle, the underlying scenario for a topological phase transition is created. If the energy level is generally low, more connections around the unhappy element will be broken, than reformed. Connections are lost, condensation occurs, and we start to go from random to scale-free, from scale-free to star and from star to disconnected subgraphs.

Topological phase transitions of networks seem to be a fairly general phenomenon. Let us consider two examples here. Some of their elements may seem rather wild, but they nicely demonstrate the imaginative power of generalization along the lines of similar network properties. Obviously, many further experiments and studies are needed to validate either of these examples.

Topological phase transition 1: Cell death

- **Random graph phase.** In this cellular state, an abundance of outside resources provokes a shift in the metabolic network towards the random graph pattern. The cell has exponential growth, low noise and uniformity.
- **Scale-free phase.** If resources are reduced and the cell experiences a low level of stress, a scale-free metabolic network develops. We have higher noise, some proteins – as elements of the cellular network – become damaged by a few perturbations, and the repair system provided by chaperones is gradually overloaded, leading to several deviant responses. Consequently, cellular diversity starts to develop (for further details, see Sects. 6.2 and 6.3).
- **Star phase.** With higher stress levels, system resources grow critical. The cell has to concentrate its energy in the form of ATP consumption for a minimal set of vital functions, and the metabolic network will shift towards the star phase.
- **Disintegration to subgraphs.** If system resources go below the critical level or noise becomes too great, too many damaged proteins develop, the system begins to disintegrate and the cell dies from apoptosis or necrosis (Sőti et al., 2003).

Topological phase transition 2: Ethology

- **Random graph phase.** The random graph phase corresponds here to parallel cooperation between members of the animal group. Every animal does the same thing and there is no sharing out of jobs and tasks.
- **Scale-free phase.** As resources are reduced, a sudden topological phase transition occurs towards complementary cooperation, and task distribution develops between various members of the animal community (Le Comber et al., 2002; Theraulaz et al., 2002).
- **Star phase.** If system resources become close to critical or the level of stress increases (e.g., carnivores arrive in great numbers), a star-phase network develops with a dictator, an alpha male (Hemelrijk, 2002).

- **Disintegration to subgraphs.** Extreme danger disorganizes the larger network, and subgraphs – core families – try to escape and survive together.[21]

Topological phase transition of prebiotic networks. Shenhav et al. (2005) described the idea that prebiotic catalytic networks might have a random configuration and raised the question as to how this early molecular ensemble changed to the scale-free metabolic networks we observe today. A topological phase transition may give a clue here. As the number of prebiotic networks grew and system resources became sparse due to mutual competition, the random network may have been forced to change configuration to the scale-free degree distribution we observe today. The differences in the reports on the degree distribution of metabolic networks may also be partially derived from the different configuration of these networks under various levels of cellular stress.

Striking similarities, are they not? I will list some more examples of topological phase transitions in firms and in human society later. But let us stay with our perturbations for a while. I have shown above that they help topological phase transitions by local disintegration of the old connections around the unlucky, perturbation-hosting elements. I also raised the point that with low system resources, not all connections will be reformed. However, I did not ask which of the links would be lost: the strong links or the weak links?

The parsimony principle would require the strong links to drop, since they are more costly. However, if the energy is lower, we need condensation. This purpose is better served by strong links. As supporting evidence, in the case of high unemployment or any other type of increased everyday stress, weak links are broken and people tend to rely on strong links (Granovetter, 1983). In the case of low system resources, implying a large environmental stress, the link strength and degree distributions may condense in parallel. The network sheds all non-essential weak parts and a core of strong links remains. The scale-freeness of degree and link strength distributions is born and becomes lost in parallel.

3.5 Nestedness and Stability: Sync

Networks are rather resistant to attack by perturbations. As we have seen in the previous sections, relaxation, topology-based resistance

[21] I am grateful to Péter Száraz for these ideas.

against network damage, and as a last resort, reconfiguration of the network topology, all contribute to network survival, e.g., the preservation of the giant component, if trouble arrives. In terms of network properties, fast relaxation requires small-worldness so that perturbations can reach the elements of the local network region as fast as possible. Confinement of perturbations to this local network region and resistance against random damage require scale-freeness. Only one important network property has been left out so far: nestedness.

We have already noted several times that we run into trouble if the perturbation gets arrested somewhere in the net. It is time to ask why we then run into trouble. This is where we need nestedness to provide an answer. If the perturbation gets stuck at a given element, the bottom network that this element actually represents will be overwhelmed by the large amount of energy it receives, since these energy packets usually just go very quickly through this particular element.

Is a transient perturbation inefficient? Why is a bottom network not affected by a large energy packet, if the packet goes quickly though this bottom network?

There appears to be a delay in the response of bottom networks, leading to a form of 'laziness', or the idea of a time window for escape, that have hitherto remained uncharacterized. During this time they can put up with the presence of the perturbation. However, if the perturbation stays any longer, the bottom network will be in trouble. One quite widespread form this trouble can take is that the affected bottom networks decouple from a synchronous oscillation. *"What do you mean when you speak of the synchrony of oscillations?"*

Oscillation synchrony was discovered by Christian Huygens when he was sick in 1665. As he lay in bed, he watched his two newly constructed pendulum clocks on the wall. He had invented these clocks himself to win the fabulous prize offered by the Royal Society in London for a clock which would be precise enough to keep time on ships sailing long distances across the sea. *"With all due respect to all their noble deeds, these Brits are a bit crazy. Did they pay such a tremendous amount of money just to have their five o'clock tea exactly on time aboard their ships?"* No, Spite, this was not the case. If you know the Greenwich mean time (or any other time at a given point on Earth) exactly, you can work out your longitude from the time difference. The British sailors needed an exact time to know where they were, and to situate the treasures they found.

But let me return to Huygens. We left him sick in bed. Well, he is still sick, and actually gets much sicker. As he lay there, he observed that the two clocks he had on the wall gradually became synchronized and finally oscillated together. Now that is interesting! Forgetting about being sick, he jumped from bed and started to reposition the clocks on the wall. After many experiments (which probably prolonged his sickness by delaying the resynchronization of the oscillators inside his own body), he realized that it was the wall that was transmitting weak signals from one clock to the other to cause the miraculous synchronization. He was proud to report his findings to the Royal Society (Huygens, 1665). The honourable members discussed his new experiments at their next session of 8 March, and arrived at an unexpected conclusion: "Occasion was here by some of the members to doubt the exactness of the motion of these watches at sea, since so slight and almost insensible motion was able to cause an alteration in their going" (Minutes of the Royal Society meeting, 8 March, 1665). Huygens never got any money for his pendulum clocks. The million dollar prize had to wait a hundred years. Huygens got very disappointed and never dealt with the synchrony phenomenon again (Strogatz, 2003).

Fortunately, others did. Steven Strogatz gives a wonderful survey of the synchronization of various oscillators in his excellent book *Sync* (Strogatz, 2003).[22] Chemical oscillators like the 64 nickel electrodes of Kiss et al. (2002) can vary their potential in synchrony; temporarily synchronized neurons ensure successful memory formation (Fell et al., 2001); and our level of synchronization with circadian rhythms (Ogle, 1866) can be conveniently studied on sleepless nights after a transcontinental flight. Visual and acoustic interactions make fireflies flash (Buck, 1938), crickets chirp (Walker, 1969) and audiences clap (Néda et al., 2000) in synchrony. Women living together synchronize their menstrual cycle (McClintock, 1971). Hare and lynx populations of Canada are even better. Their population cycle can manifest a synchrony over millions of square kilometers (Blasius et al., 1999). As a less glorious, but equally large-scale example, the occurrence of syphilis was synchronized in the whole of the United States from New York City to Houston between 1960 and 1993 (Grassly et al., 2005). As a final example, unconscious synchronization of fine movements during the steps of the celebrating crowd caused a violent wobbling of the 690 ton London Millennium bridge on 10 June 2000, on the day of its

[22] Synchrony is also called entrainment, especially in music.

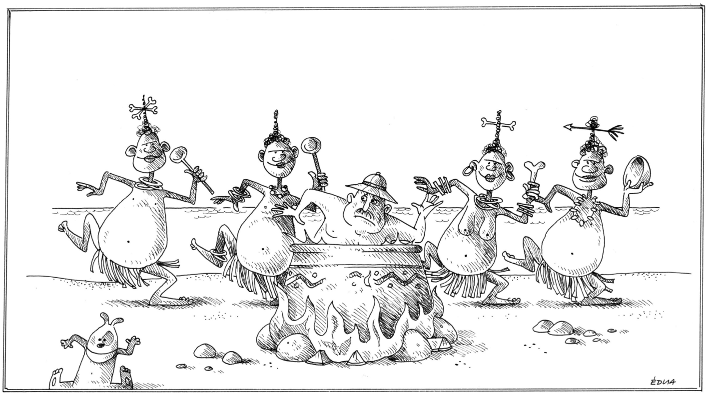

Fig. 3.9. Synchrony seems to be a joyful phenomenon. It tends to make us feel stable and safe

opening (Strogatz, 2003). In spite of the last few examples (and Huygens' despair), synchrony seems to be a joyful phenomenon. It tends to make us feel stable and safe (where 'us' refers to fireflies, crickets, hares, lynxes and even nickel electrodes – a fine brotherhood, indeed).

Music and learning again. Is the beneficial effect due to synchronicity? You may remember my remark in Sect. 3.1 that listening to the right kind of music – I am not referring to quality here, but simply requiring that it has a pink-noise-type, scale-free structure – may help your learning abilities. Here I would like to put forward the idea that the scale-free pulses of the external noise may help neuronal synchronization, which is responsible for memory formation (Fell et al., 2001). Next time you forget something and start to scratch your head, think about this. External noise may help your internal oscillations.

Synchronization is a network phenomenon, not only in the sense that it requires a network of oscillators to happen, but also in the sense that it has many properties characteristic of networks. Here are two examples from among many:

- Similarly to the percolation threshold of networks, synchronization also has a phase transition. As the difference between the frequencies of different oscillators is decreased below a certain threshold, all of them will suddenly become synchronized, achieving syntalansis (see Fig. 3.10) (Winfree, 1967). Synchronization has nestedness. Network elements at all levels may behave as synchronized oscilla-

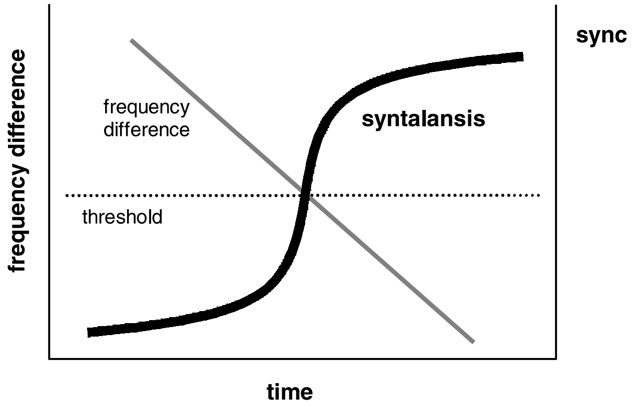

Fig. 3.10. Achieving syntalansis, the sync phase transition. The phase transition of synchronization is shown schematically here. As the difference between the frequencies of different oscillators decreases below a certain threshold, they are all suddenly synchronized, achieving syntalansis (Winfree, 1967). The *grey line* denotes the frequency difference, while the *bold curve* illustrates the relative extent of synchrony

tors. Cells are synchronized like the neurons in our brain to make recognition possible. Organs are synchronized to help lampreys and leeches to swim as well as to produce the peristaltic movement helping our digestion. Finally, organisms are synchronized with each other (Bressloff and Coombes, 1998; Strogatz, 2003). However, synchronization also has modularity, which means that the synchronized state may only extend to some of the networking oscillators (Winfree, 1967).

- Synchronization has self-organized criticality. The roughly 10 000 pacemaker cells in the sinoatrial node of the heart can be efficiently modeled by a network of oscillators, where each of the oscillators gradually increases its membrane potential and then, after reaching a threshold, becomes discharged. When the discharge occurs at any particular oscillator, all neighboring oscillators will become slightly more depolarized. Mirollo and Strogatz (1990) showed that these oscillators produce an avalanche as they subsequently become synchronized with one another. The final phenomenon is very similar to earthquakes or other avalanches discussed in Sect. 3.3. The extent of synchronization is also important, since disruption of local synchronization prevents efficient relaxation and may cause self-organized criticality and subsequent avalanches (Ponzi and Aizawa, 2000).

Synchronization depends on the network properties. Everything making the network better connected helps synchronization. The seminal paper by Watts and Strogatz (1998) showed that small-worldness helps synchronization. Actually, small-world topology is extraordinarily effective for this purpose (Barrahona and Pecora, 2002). Small-worldness reduces the divergence between extremes of the individual oscillators, and this helps to preserve the synchronized state (Guclu and Korniss, 2004).

Scale-freeness of the top network has a rather adverse effect on the extent of synchronization. Hubs connecting many nodes to each other have to be avoided, since these 'center' oscillators interacting with a large number of other oscillators tend to be overloaded by the traffic of communication passing through them (Nishikawa et al., 2003). When the hubs of a scale-free network of oscillators were replaced by triads, the level of synchronization was remarkably increased (Zhao et al., 2005). Assortative networks, where hubs are coupled to hubs, were found to synchronize even less well due to the mutually disturbing effects of the overloaded hubs (di Bernardo et al., 2005). As we have seen in previous sections, scale-free structure tends to confine the effects to smaller segments of the network. Here again, small-worldness and scale-freeness interact to keep a balance of optimal synchronization.

Jung revisited: A possible example of nested sync. The former remark on nestedness brings me to the various and sometimes rather vague interpretations of sync. In the famous essay by Carl Jung (1969), synchronicity was perceived as the opponent of "constant connection through effect" (causality) and meant an "inconstant connection through contingence, equivalence or meaning". Among the many examples Jung gave, I would only list here fulfilled dreams and prayers.[23] Thinking about synchronicity in the context of the present chapter, i.e., the synchronicity of oscillators and networks, many of the Jungian examples can actually be interpreted, if we suppose that strong synchronization in a given network may induce a synchronization of the elements of the nested network one level up or down (called here nested sync). Thus the fulfilled dream Jung mentioned on a disaster in a remote island may be perceived as synchronization propagation from the top to the bottom network. Here a synchronization event in the top

[23] I would like to note here that most of the other examples, e.g., Jung's extensive experiments on astrology, are much more difficult to accept in the present context of nested sync, and in the current state of scientific knowledge.

network (the simultaneous death of several people on the island) may induce the synchronization of the elements (neurons) of one of the bottom networks (the dreaming person) and cause the dream.[24] Conversely, a fulfilled prayer may be perceived as a synchronization propagation from the bottom to the top network. A high level of synchronization of the neurons in one of the bottom networks may cause a synchronization of the top network and the prayer becomes fulfilled.[25] What do we call the top network in this case? Well, it depends on your religion! I think, this is the point where I have to remind you that we are in a triple smiley box, which means that the content is fiction and not science. However, I have one further remark: an increased coherence of electroencephalograms (EEGs) has been repeatedly observed during meditation (Aftanas and Golocheikine, 2001; Orme-Johnson and Waynes, 1981). This is by no means a proof for nested sync (no one is able to measure the EEG-like phenomenon one or two networks higher!). However, it shows that intensive mental states can indeed lead to more coherent brain functioning. My last remark is about Mozart. Did the neurons in Mozart's head know that they were in sync because the Master had just conceived the Requiem? Moreover, did the Master know that the Requiem would be in sync with his own death? It is a bit difficult to grasp trans-network connections, especially from the bottom network. We have to be patient with the explanations given by Mozart's neurons. I hope I have convinced you that you also have to be patient with my explanations.

My first encounter with nested sync. *"Peter, if you start talking about your fulfilled dreams here, I'm leaving."* Do not worry, Spite, *this* dream of mine will never be fulfilled! I just want to offer an explanation as to why I might be so sensitive to the above, rather unusual ideas. When I was around four, I had a terrible ear inflammation. Suffering unbearable pain, an idea suddenly came into my mind: the fact that I had to suffer had a purpose. Something much bigger was using me to have good ideas, and it was the unusual intensity that was giving me the pain. I agree that this may be even less well grounded than any of the ideas in the comment above, but it worked! When I got to this point, the pain had stopped.

So far I have outlined some of the possibilities for the way synchrony develops. We have seen that network properties like small-worldness

[24] If the fulfilled dream is about a future event, well, let's keep the 'explanation' for my next book!

[25] Actually, it would be very interesting to measure the difference in the synchronization intensity of the affected neurons during an ordinary and an emotionally involved, intensive prayer.

and scale-freeness assist synchronization. We even had an esoteric example of nestedness in sync. However, in all these examples, sync was either there or it was not. The question arises as to whether there are levels of synchronization? Well in fact there are. We have basically three levels. If synchronization is weak, the frequencies will be uniform. At the next level, the phases of the waves will also be synchronized. At the tightest coupling level, the amplitudes of oscillations will also be the same. How are the various levels of synchronization achieved? To a first approximation, we should remember Huygens and his wall. Indeed, there must be an interaction between the individual oscillators to get them synchronized. How strong should this interaction be? *"I guess we will see your weak links again."* Yes. Fasten your seat belts! The weak links are on their way.

Strong links lead to amplitude synchrony, while weak links induce only a weaker, phase synchronization (Blasius et al., 1999). *"Well Peter, your weak links seem to lose out this time."* There is no need for that glorious smile, Spite. Weak synchronization intensity does not mean weak benefits. Several famous models of synchronization, like the Josephson effect in superconductors, the Winfree model, or the Kuramoto model use weak links to achieve synchrony (Ariaratnam and Strogatz, 2001; Feynman et al., 1965; Kuramoto, 1984; Strogatz, 2003; Winfree, 1967). Oscillators are not usually identical and their coupling is therefore weak. As an example of this, individual hamster clock cells are very diverse, and their coupling is weak. Hamsters still have a rather good circadian rhythm (Liu et al., 1997). Moreover, stochastic resonance[26] of coupled oscillators is best achieved if oscillators are coupled by weak links (Gao et al., 2001; Lindner et al., 1995; 1996).

The Winfree and Kuramoto models: Examples of weak-link-induced sync. The Winfree model describes the synchronization of N coupled phase oscillators with the formula

$$\theta_i = \omega_i + \frac{\kappa}{N}\sum_{j=1}^{N} P(\theta_j)R(\theta_i) \ ,$$

where $\theta_i(t)$ is the phase of the ith oscillator, κ is the coupling strength, and the frequencies ω_i are drawn from a symmetric unimodal density $g(\omega)$. The model assumes that the mean of $g(\omega)$ equals 1. $P(\theta_j)$ is the influence function of the jth oscillator, while $R(\theta_i)$ is the sensitivity function of the

[26] In stochastic resonance, noise helps in the detection of sub-threshold signals. For a proper description of the phenomenon, please turn back to Sect. 3.1.

i th oscillator with respect to the average influence of all oscillators (Winfree, 1967). The Kuramoto (1984) model is a refined model of the Winfree model. Here the same N coupled oscillators are described by the formula

$$\theta_i = \omega_i + \sum_{j=1}^{N} \kappa_{ij} \sin(\theta_j - \theta_i) \ ,$$

where the symbols denote the same as above. Both models display synchrony at relatively low coupling strengths κ.

Proper weights improve the synchronization of scale-free networks. As mentioned earlier, networks with scale-free degree distribution generally disturb synchronization. However, if the scale-free degree distribution is accompanied by a scale-free weight distribution, where the weights are distributed in such a way that all the nodes receive the same input signal, the synchronization of scale-free networks is greatly improved. In fact, this weight distribution corresponds to the minimum cost, which is related to the total strength of all directed links (Motter et al., 2005).

In summary, we may conclude that there are many examples in which weak links couple individual oscillators and help them to develop synchrony. Is there any special benefit from synchronization in general, and weak-link-induced synchronization, in particular? The early work by Enright (1980) already put forward the idea that coupled oscillators are more stable. Later it turned out that an optimal level of sync is the best to achieve the highest stability (Yao et al., 2000). If we have complete sync by strong links, we have lower stability than with partial sync by weak links.

Analogy between the optimal level of sync and the notion of local dissipation and global connection. The main message in Sect. 2.2 was that properly designed networks should keep a balance between rather free local traffic to ensure the proper dissipation of perturbations and global connectedness to help system-wide responses of the whole integrated network. This idea is strongly analogous to the optimal level of synchrony we observe here. Complete synchrony would help the propagation of perturbations in the whole network, leading to a continuous risk of cascading failures. Zero synchrony would isolate various segments of the network from each other and prevent system-wide responses.

How is the ‘extra stability’ of partial sync achieved? For the answer, let us turn back to the starting point, i.e., to the perturbations. Strogatz et al. (1992) showed that, in cases where the frequency distribution of the oscillators is restricted to a frequency interval, the dissolution of synchrony after a perturbation is slower than exponential. In other words, partial sync is better for sticking oscillators together. If the perturbation arrives disguised as a traveling wave, only those oscillators stay coupled which were coupled weakly (Bressloff and Coombes, 1998). As a third example of the same phenomenon, phase synchrony, the specialty of weak-link-induced synchronization, seems to be particularly resistant against the attacks of various perturbations (Blasius et al., 1999).

Weakly coupled oscillators seem to relax faster. Obviously, if the coupling is too weak, it becomes ineffective in the sense that extremely weak coupling between oscillators gives no synchrony at all. However, weak coupling seems to be quite profitable. If you want to get a stable joint oscillation, a Winfree syntalansis (1967), then seek weak links. The following advice emerges: stay away from authoritarian groups requiring full synchrony. In Sect. 10.3, I describe in detail the study by Kunovich and Hodson (1999), who showed that after massive stress (the civil war in Croatia), psychic recovery was better helped by informal organizations than formal ones. The explanation is rather straightforward: informal organizations allowed better sync and faster relaxation of the prolonged tension.

Why do we like sync? Synchronization certainly gives us pleasure. To be honest, I have no idea whether the nickel electrodes, our neurons, the fireflies or the crickets were happy producing synchrony. Moreover, it is difficult to measure whether groups of women feel more secure emotionally when their menstrual cycles are synchronized. However, when audiences clap in unison (Néda et al., 2000), people make waves in stadiums (Farkas et al., 2002), a crowd sings the national anthem, people laugh, or I keep my knees hitting in the same rhythm as my dog runs when we are out together in the park, there is no doubt that all these things give great pleasure.

Synchronized laughter quakes. Laughter is a rather joyful act. We spend countless hours of our life in search of laughter and involved in attempts to make others laugh (Dunbar, 2005). We are not alone. Even rats laugh – though in the ultrasound region (Panksepp and Burgdorf, 2000). I would like to put forward the idea that laughter is in fact a synchronized self-organized criticality phenomenon. Before we start to laugh, there

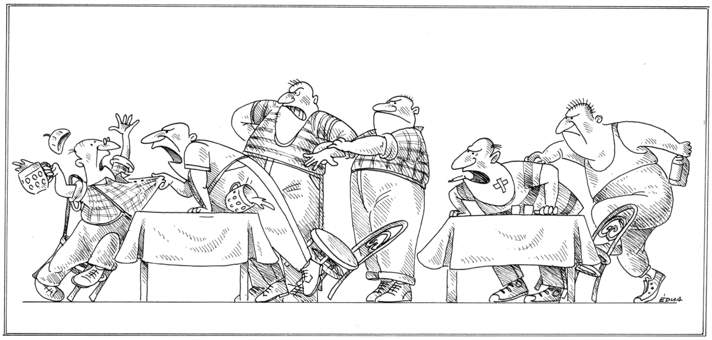

Fig. 3.11. Tensions that any of us are unable to cope with alone are transferred to others by the sync we all enjoy

is a gradual input of information causing tension, which finally bursts into laughter. As an example of the gradual input, think about the 'annoyance' of repeated unlikely events, or the cognitional tension of an outright absurdity. Laughter is certainly a relaxation phenomenon. It would be nice to see whether the intensity and duration of laughter display scale-free behavior. However, we seldom laugh alone. Laughter is provoked by others' laughter in a rather contagious fashion. Laughter is a very good example of our inherent love of sync.

Why do we like sync? Sync may give us an additional level of stability by helping relaxation. Fast relaxation stabilizes systems and sync helps relaxation. Optimal sync helps local relaxation. Relaxation has been conditioned to cause joy so that we learn to seek it.[27]

Let me ask you to take a deep breath, to drink a glass of crystal clear water, to relax, and most importantly, to think. Is this not beautiful? For our life we need a continuous inflow of energy. However, this generates ever more tensions, which may cause our death. Relaxation is indeed a question of life or death for us. This is a formidable task, but we are not alone! We are in sync. This gives a safety net to all of us. The tension that any of us is unable to cope with alone is transferred to others by the sync we all enjoy. The most beautiful moments and feelings of our life: joy, laughter and happiness are all tied to relaxation and sync. This is our reward, if we successfully learn about one of the most general laws nature has yet made for collective survival.

[27] Relaxation comes only after the sync has been set and an unexpected perturbation is experienced. Therefore it is better to be conditioned by another feeling, joy to seek the sync, which will ease future relaxations. I will return to the connections between joy, relaxation and stability in the concluding chapter of the book.

"Peter! Don't you think it's time to put your ideas into practice and stop writing NOW. Then you can start that well-deserved relaxation?" Spite, it is truly kind of you to be so considerate about my health, but now you have spoilt a great moment for me. Here I am, sitting with a glass of crystal clear water in front of me, with my soul in heaven, and you drag me back down to Earth. But don't be sad, Spite. You have only done what you are supposed to. Returning to your question, believe me, there are so many exciting thoughts in my mind that if I stopped writing NOW, I would end up in the madhouse and never achieve any form of relaxation. Writing is my relaxation right now, and I thank the publisher and the reader for giving me this opportunity.

The emergent property of human sync. Superconductivity, heart beat, peristaltic movements and our thoughts are all emergent properties of sync at the level of the top network. We can recognize the emergent property if network members are particles or cells. What is the emergent property when we humans start to make sync? Do the millions of synchronized exclamations after a goal in the final of the world soccer championship mean that Gaia has remembered a great joke she heard in the Cambrian epoch?

In the last few sections we have learned several recommendations about how to save our networks. We have seen how to discriminate between good and bad noise. We now recognize the degree of danger if a perturbation arrives alone or with a number of other perturbations. We have learned how to confine relaxation to a segment of the network. We have studied avalanches, network failures, topological phase transitions and finally sync. How should we put all this into practice? How can we construct a network which will resist against all perturbations? Stay tuned! The next section will try to give a clue.

3.6 How Can We Stabilize Networks? Engineers or Tinkerers

Network design and network stabilization are engineering tasks. Stability can be achieved in connected systems by negative feedback, for example, a typical element of engineered systems. Indeed, highly optimized engineered systems display a considerable level of stability. These systems have many highly optimized feedback regulations called integral feedback control or more generally, highly optimized tolerance

(HOT). Modern machines demonstrate a very high level of organization. In a Boeing 777, a hundred and fifty thousand different subsystem modules can be found, piloted by close on a thousand computers. The final testing of such an airplane generates data almost equivalent to the human genome every minute (Carlson and Doyle, 2002; Csete and Doyle, 2002).

However, our sophisticated machines were not the first complex systems on Earth to show this level of stability. We ourselves are also pretty good examples of complex and stable networks, and date back somewhat further in evolution than the Boeing 777. Francois Jacob (1977) pictured evolution as a tinkerer, "who does not know exactly what he is going to produce, but uses whatever he finds around" and "gives his materials unexpected functions to produce a new object". Indeed, evolution does not optimize the system in advance making a blueprint, but assembles interactions until they become good for the task (Maynard-Smith and Szathmary, 1995). Steven Rose put the same idea in his *Lifelines* (1997): "We carry the burdens of the past with us."

What are the common features of the engineered and evolutionarily tinkered systems? Just to name a few of the most important ones: modularity, robustness, and on the other side of the coin, failure avalanches (Carlson and Doyle, 2002; Csete and Doyle, 2002). However, there are major differences between the results of engineering and tinkering:

- **Evolution must make all intermediates viable.** As opposed to an evolutionary system, an engineered system has been optimized for the purpose at hand. In engineering, there is no need to optimize all predecessors and there is not such a strict requirement for continuity amongst these predecessors. Finally, the engineered system is not forced to change by introducing just a few small changes at each point in its development. Though the concept of punctuated equilibrium (Gould and Eldredge, 1993) introduced discontinuity into the evolutionary process, and later a few molecular mechanisms (Rutherford and Lindquist, 1998; see Sects. 6.2 and 6.3 for details) were also uncovered to explain jumps in evolution, the required level of continuity still makes a difference between engineered and evolutionary systems.
- **An engineered system is complicated, while an evolutionary system is complex (Ottino, 2004).** In the case of complicated systems, the pieces can be disassembled and reassembled again, and the function of the whole can be guessed quite well from the functions of the parts. In the case of complex systems (for a

discussion of complexity, see Sect. 4.3), the function of the whole is an emergent property of the parts, and in most cases we cannot make a straightforward guess about the function of the top network if we only know the function of the bottom networks (modules) in a piecewise manner.

- **As opposed to engineered systems, evolutionary networks are integrated and their parts cannot be optimized separately.** In their famous essay against the Panglossian Paradigm, which considers each part of a complex organism as the result of evolutionary optimization, Gould and Lewontin (1979) wrote: "Organisms are integrated entities, not collections of discrete objects." Although engineered systems are also integrated, the function of individual parts in these systems is better described and this function is usually close to optimal by itself.
- **Evolutionary networks have greater designability.** Evolutionary networks have a higher capacity for combinatorically different setups of their components than engineered networks (Changizi et al., 2002). This property is also called designability (Tiana et al, 2004).
- **The evolutionary design is stable under many conditions.** As a result of the design process, engineering gives stability only with very finely tuned parameters, while evolutionarily tinkered networks are stable under a much wider range of initial conditions (Aldana and Cluzel, 2002).
- **Link strength difference between engineered and evolutionary systems.** As shown in Sect. 2.4, evolutionary systems develop a continuous range of interaction strengths between their parts. In engineered systems, reliability is a crucial factor. Two parts either interact, or they don't. Probabilistic, vague, 'almost' interactions do not reflect a skillful design. Though interaction strength can be defined by the duration of interaction, and in this way engineered systems also contain weak links, link strength diversity is rather limited in engineered systems as compared to evolutionary networks.
- **The evolving system grows.** Finally, an engineered system does not necessarily grow, whereas the evolutionary system grows by definition.

What happens if a self-organized system cannot grow? Is growth arrest a source of aging and death for networks? Is growth arrest a serious form of stress which leads to a series of topological phase transitions of the network (see Sect.3.4), resulting finally in the disintegration of the net and death? Are we sentenced to grow in running away from our own death?[28]

Evolution can go backwards. Originally the above list had an additional point: it was thought that, in contrast to engineering design, evolution cannot restore information that has already been deleted. In a way this statement was not true even at that time, since the deleted genetic information might have been stored in another species, in such a way that it could be regained. However, the recent paper by Lolle et al. (2005) has described an even more elegant way to save the blueprint of a good old design, until the new one proves that it is indeed better. The blueprint is actually pale-blue here, since the old and discarded genetic information is saved, not in the form of DNA, but in the form of RNA. The reversion of the particular gene in the plant *Arabidopsis thaliana* to the older and discarded version was as high as 10% in some cases. The process was governed by an RNA segment which was retro-transcribed to the DNA. The mechanism may actually be quite general. However, many more experiments are needed to assess the importance of this non-Mendelian inheritance.

Although I have shown that engineered and evolutionary developed systems differ in many respects, the contradiction between the engineer and tinkerer types of development is only apparent. In Sect. 9.5, I show the convergence of the two developmental schemes in highly sophisticated, modern designs.

[28] I am grateful to Bálint Pató for these questions, which are related to the necessity of housekeeping heat (Oono and Paniconi, 1998) mentioned in Sect. 3.1.